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SAMPLING PLANS BASED ON THE WEIBULL DISTRIBUTION*

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Henry P. Goode and John H. K. Kao
Department of Industrial and Engineering Administration
Sibley School of Mechanical Engineering
Cornell University, Ithaca, New York

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SUMMARY

This paper presents a proposed set of acceptance-sampling plans for life testing and reliability when the underlying life distribution is of the Weibull form. Inspection of the sample is by attributes with the life test truncated at a preassigned time, t. A set of conversion tables is also provided from which attribute sampling-inspection plans of any desired form may be designed for the Weibull model or from which the operating characteristics of any given plan may be determined. A procedure using these tables for applying the MIL-STD-105B plans to reliability and life-testing applications is included.

INTRODUCTION

The paper is a generalization of papers by Sobel and Tischendorf[1] and by Epstein[2] that appeared respectively in the Proceedings of the Fifth and Sixth National Symposium on Reliability and Quality Control in Electronics. Related work has also been done by Gupta and Groll[3] who have extended the Sobel and Tischendorf procedures from the exponential form to the gamma form. The gamma variable is the sum of exponential variables and hence the exponential model is a special case of the gamma. In the two papers first cited the authors assume that the underlying life density is exponential (Eq. 1) whereas in this paper the Weibull form is assumed (Eq. 2). The exponential distribution is a special case of the Weibull and so will be covered in the plans and conversion tables.

$$f(x) = (1/\mu) \exp [-x/\mu], \ \mu > 0, \ x > 0 \tag{1}$$

$$f(x) = (\beta/\eta)(x/\eta)^{\beta-1} \exp [-(x/\eta)^{\beta}], \ \eta > 0, \ \beta > 0, \ x > 0 \tag{2}$$

Both f(x) are equal to zero, otherwise. In these and the equations that follow, X is a random variable which represents item life for which x is its value, μ represents mean item life for the population, and β is the symbol for the shape parameter for the Weibull distribution. For simplification in discussion and computation, the characteristic life, η has been used. For the Weibull distribution

$$\eta = \mu/\Gamma(1/\beta + 1) \tag{3}$$

For further discussion of the Weibull distribution as a statistical model for lifelength of components or systems, reference may be made to a paper by Kao[4] in the Proceedings of the Sixth National Symposium on Reliability and Quality Control in Electronics.

From Eq. (3), it will be noted that for a given η the Weibull mean life depends on the shape parameter, β. The effect of differences in the parameter on the shape of the distribution, as well as the general nature of the Weibull model, may be observed by study of Figure 1. In this figure the Weibull probability density function has been plotted for various value of β. A plot for $\beta = 1$, the exponential case, has been included for reference. This initial set of Weibull sampling plans is for product for which the value of this parameter is known or can be assumed to approximate some given value. Conversion tables and sampling plans are provided for nine values for β ranging from 1/3 to 5.

A relatively small number but a broad range of values for β was selected for this initial study. The principal objectives were to develop practical methods and techniques and to explore the effect of differences in value for this parameter (which is the key one for the Weibull distribution). As the use of this distribution as a statistical model increases, additional conversion tables and sampling inspection plans may be constructed for intervening values for β, particularly in the widely encountered region ranging from 1/2 to 2.

For the procedures and plans developed in this study, lot or product quality is evaluated in terms of mean item life, μ. Subsequent work has been planned in which related conversion tables and sampling inspection procedures will be developed for application when lot quality must be evaluated in terms of the instantaneous failure rate, $Z(t)$, at some specified life or future time, t.

FORM OF ACCEPTANCE CRITERIA

For the plans considered in this paper, the following acceptance-sampling procedure for life testing has been assumed:

1. Select a random sample of n items from the lot.

2. Put the sample items to life test for some preassigned period of t time units.

3. Denote by y the number of failures observed prior to time t.

4. Accept the lot if $y \leq c$, the acceptance number; if $y > c$, reject the lot.

Curtailed inspection for accepting lots prior to t is possible for the rejection of the lot since it is possible to observe $(c + 1)$ failures before time t.

Note that this acceptance procedure is the same as that specified for the MIL-STD-105B[5] sampling plans with the exception of the introduction of a testing truncation time, t. It is also possible (as for the 105-B plans) to employ double or multiple sampling instead of single sampling as described above and by so doing reduce the average number of items at $p' \cong$ AQL that must be put on life test. However, the "economy" achieved is at the expense of longer elapsed testing time.

The probability of acceptance for a lot, P(A), under plans of the above form depends on the probability, p', of item life being less than (or equal to) the test truncation time, t. For cases for which β is known and with time, t, preassigned, p' is thus a function of mean item life, μ, only. The operating characteristics of any specified plan thus depend only on t and μ. In order to provide tables for general use in the design or evaluation of plans for any application rather than working in terms of specific values for t and μ, the dimensionless ratio t/μ will be used. In application of the plans or tables to a specific application, conversion between the ratio and specific t and μ values is extremely easy.

A set of conversion tables has been computed to provide for the Weibull distribution the connection between the dimensionless quantity t/μ and p' (Tables 1 and 2). With these tables acceptance-sampling plans of desired form can be designed or evaluated using attribute sampling theories and practice.

For cases for which the lot size, N, is large in relation to the sample size, n, the number of failures prior to t approximates the binomial distribution with parameters n and p', where p' is defined as the area under the life-length distribution up to t. The probability of acceptance P(A) depends on the cumulative number of failures prior to time t. This probability is given by

$$P(A) = P\{y \leq c\} = \sum_{y=0}^{c} \binom{n}{y} p'^{y} (1-p')^{n-y} \tag{4}$$

The binomial distribution has been assumed for the sampling plans given in this paper except for cases for which the sample size is relatively large. For these, the Poisson distribution has been used as an approximation to the binomial. The probability of acceptance for the Poisson is given by

$$P(A) = P\{y \leq c\} = \sum_{y=0}^{c} \frac{(np')^{y}}{y!} e^{-np'} \tag{5}$$

An important potential use for the conversion tables provided in this paper is in the adaptation of the MIL-STD-105B plans to reliability and life-testing applications. In describing the operating characteristics of these 105B plans, the quality of submitted lots is measured in terms of p', the per cent defective. With the conversion factors this form of description may be converted directly to measurement in terms of the t/μ ratio. With this conversion the 105B plans may be cataloged for appropriate choice in reliability applications. (Plans have been made for the preparation of tables listing for various values of β this ratio at the AQL and the LTPD for each plan in the 105B manual.) Alternatively, if some 105B plan has been selected, its operating characteristic curve may be determined in terms of the t/μ ratio, or if the testing time, t, has been specified, in terms of the lot mean, μ. An example employing such a conversion is shown later in this paper. It should also be noted that with the matching plans provided in the 105B collection, the options of double-sampling and multiple-sampling are also available. The sample sizes and acceptance numbers listed may be used and the established procedures for employing this form of sampling in attribute inspection may be followed.

COMPUTATION OF CONVERSION TABLES

The probability, p', of an item failing before the end of test time t is given by the cumulative distribution function (c.d.f.). For the Weibull model the equation for this function is

$$\text{c.d.f.} = F(x) = 1 - \exp\left[-x^{\beta}/\alpha\right]. \tag{6}$$

The equation for the mean, μ, of the Weibull distribution is

$$\mu = \alpha^{1/\beta}\,\Gamma(1/\beta + 1)\ . \tag{7}$$

Computations may be simplified by the following substitutions:

$$b = 1/\beta \tag{8}$$

$$\eta = \alpha^{1/\beta}\ . \tag{9}$$

Equation (6) then becomes

$$F(x) = 1 - \exp\left[-(x/\eta)^{1/b}\right], \tag{10}$$

and Equation (7) becomes

$$\mu = \eta \, \Gamma \, (b+1) \quad . \tag{11}$$

The probability, p', of an item failing before the end of test time, t, is thus given by

$$F(t) = 1 - \exp\,[-(t/\eta)^{1/b}] = p' \; . \tag{12}$$

This may be rewritten as

$$1/(1 - p') = \exp\,[(t/\eta)^{1/b}] \quad , \tag{13}$$

which in turn may be converted to

$$- \ln\,(1 - p') = (t/\eta)^{1/b} \tag{14}$$

where ln denotes the Naperian logarithm.

Raising both sides of the above equation to the b power gives

$$[-\ln(1-p')]^{b} = t/\eta \; . \tag{15}$$

This equation may be solved for η to give

$$\eta = t/[-\ln(1-p')]^{b} \quad . \tag{16}$$

But from Equation (11),

$$\eta = \mu/\Gamma(b+1) \quad . \tag{17}$$

Substitution of this value for η in Equation (16) gives:

$$\mu/\Gamma(b+1) = t/\;[-\ln(1-p')]^{b} \; , \tag{18}$$

or $$t/\mu = [-\ln(1-p')]^{b} \,/\, \Gamma(b+1) \; . \tag{19}$$

This equation establishes the relationship between the dimensionless ratio t/μ and p', the probability of item life being equal to or less than t.

It may be noted that for the attribute form of sampling inspection considered here, only this dimensionless ratio between test time, t, and item mean life, μ, need be of concern. The Weibull scale parameter, α (or its equivalent characteristic life, η) has been eliminated. In the mathematics of these plans and procedures it has been assumed that the Weibull location parameter, γ, has a value of 0. If in application, however, γ has some non-zero value, all that is necessary is to subtract the value for γ from the value for t to get t_o, and from the true lot mean μ, to get μ_o. These converted values, t_o and μ_o, are then used for all computations. Any solutions in terms of t_o or μ_o can be readily converted back to real or absolute values by simply adding the value for γ. This procedure for handling the location parameter will be illustrated later in Example 3. Only the parameter β (or b, which is $1/\beta$) must be known.

To put this relationship equation (Eq. 19) in a form for which numerical values for relationships may be more easily computed, the following change may be made:

$$t/\mu = [-\ln(1-p')]^b/\Gamma(b+1) = \exp\{\ln[-\ln(1-p')]^b\}/\Gamma(b+1)$$

$$= \exp\{b \ln[-\ln(1-p')]\}/\Gamma(b+1) . \tag{20}$$

Values for the expression

$$\ln[-\ln(1-p')] \tag{21}$$

were obtained from a table of the inverse of the cumulative probability function of extremes prepared by the National Bureau of Standards.[6] This table tabulates the function

$$y = -\ln(-\ln \Phi_y) . \tag{22}$$

By substituting $(1 - p')$ for Φ_y the negative value of Expression 21 is obtained. Values for e raised to this power were read from the National Bureau of Standards tables of the exponential function.[7] Values for the gamma function, $\Gamma(b + 1)$, were obtained from a table prepared by Dwight.[8]

A table of values for the per cent truncation, $(t/\mu) \times 100\%$, for various values of p' has been prepared. It is presented as Table 1. Values for p' range from .010% to 80% with the tabulated values selected in accordance with a standard preferred number series. For convenience in both tabulation and use, both the ratio t/μ and p' are expressed as percentages rather than decimal ratios.

For determining without interpolation the value for p' when some rounded value for the $(t/\mu) \times 100$ ratio is given, the relatively simple task of preparing a table of p' has been carried out. Referring to previous equations it was noted that

$$t/\mu = [-\ln(1-p')]^b/\Gamma(b+1) \ . \qquad \text{Eq.}(20)$$

Raising each side of this equation to the β power gives

$$(t/\mu)^\beta = -\ln(1-p')/[\Gamma(b+1)]^\beta \ . \tag{23}$$

From this, an expression giving the value for p' is found. It is

$$p' = 1 - \exp\{-(t/\mu)^\beta [\Gamma(b+1)]^\beta\} \ . \tag{24}$$

The table of values for p' for various values for $(t/\mu) \times 100$ is presented as Table 2. Values for $(t/\mu) \times 100$ range from .010 to 100. Again, the values used for tabulation form a preferred number series. With this alternate table available together with the basic original one (Table 1), a conversion may readily be made either way--from $(t/\mu) \times 100$ to p' or from p' to $(t/\mu) \times 100$. Also, it will be noted that the two supplement each other in that β values giving a compressed range of figures in one table give an expanded range in the other. This allows for somewhat more precise interpolation in conversion. The two together supply basic data for the design or evaluation of any life-testing and reliability sampling inspection plan based on the Weibull (or exponential) distribution and of an attribute form. For general information, the relationship between the $(t/\mu) \times 100$ percentage and p' as given by these two tables has been plotted in Figure 2 for each of the various β values.

ESTIMATION OF THE SHAPE PARAMETER

In many applications the shape parameter, β, may be known for the product in question. From past analysis of life testing results, it may be established that some value of known magnitude may be expected regularly and so may be used in sampling inspection procedures. For example, for a certain class of electron tubes of receiving type, Kao[9] has found from study of approximately 2,000 tubes of a variety of types and applications that a value of 1.7 may be appropriate. For ball bearings, Lieblein and Zelen[10] found a mean value of 1.51 with 50% of approximately 5,000 bearings tested having β in the interval 1.17 to 1.74.

For products for which the value for β is not known, this parameter must be estimated using failure data from past inspection and research. Such data may be available in either grouped or ungrouped form.

Ungrouped Data

In this case the failure data will consist of the exact life length of each of the r items that fail out of the n that are tested. These life values may be listed in order and designated by the notation $0 \leq x_1 \leq x_2 \ldots \leq x_{r-1} \leq x_r$. The method of maximum likelihood may then be applied and an estimate for $\beta, (\hat{\beta})$, obtained by solving the following equation:

$$(1/r)\left\{ \sum_{i=1}^{r} x_i^{\hat{\beta}} + (n-r)\, x_r^{\hat{\beta}} \right\} = \frac{\sum_{i=1}^{r} x_i^{\hat{\beta}} \ln x_i + (n-r)\, x_r^{\hat{\beta}} \ln x_r}{r/\hat{\beta} + \sum_{i=1}^{r} \ln x_i} \quad . \tag{25}$$

Grouped Data

For this case the failure data will consist of the numbers failing, f, during each of a series of k conveniently chosen inspection time intervals, z. This ordered paired data may be noted as $z_1, f_1; z_2, f_2; \ldots z_{k-1}, f_{k-1}; z_k, f_k$ where $z_1 < z_2 \ldots < z_k$ and where $f_1 + f_2 + \ldots f_k = r$. The maximum likelihood estimate for the shape parameter (and for the scale parameter, α, as well) are obtained by maximizing the expression

$$\frac{z_k^{\hat{\beta}}}{\hat{\alpha}}\left(\sum_{j=1}^{k} f_j - n \right) + \sum_{j=1}^{k} f_j \ln\left(e^{\frac{-z_{j-1}^{\hat{\beta}}}{\hat{\alpha}}} - e^{\frac{-z_j^{\hat{\beta}}}{\hat{\alpha}}} \right) . \tag{26}$$

For grouped data the method of minimized chi-squares may be used. Estimates for the two parameters are obtained by minimizing the expression

$$\left(n - \sum_{j=1}^{k} f_j \right)^2 e^{\frac{z_k^{\hat{\beta}}}{\hat{\alpha}}} + \sum_{j=1}^{k} f_j^2 \Big/ \left(e^{\frac{-z_{j-1}^{\hat{\beta}}}{\hat{\alpha}}} - e^{\frac{-z_j^{\hat{\beta}}}{\hat{\alpha}}} \right) . \tag{27}$$

Graphical Method

The above methods for estimation, it will be noted, are quite involved. For accuracy and economy in computation a high-speed electronic computer must be used. However, a simple graphical method for estimation of the Weibull parameters has been devised. Estimates are obtained by plotting failure data on Weibull probability paper. The method depends on the fact that the Weibull c.d.f., $F(x)$, given by Eq. (6) becomes a straight-line equation upon a double logarithmic transformation. Thus

$$\ln \ln \frac{1}{1 - F(x)} = -\ln \alpha + \beta \ln (x) . \tag{28}$$

This Weibull paper has ln ln versus ln coordinates so the c.d.f. will plot as a straight line. Convenient scales are provided for direct plotting of raw data and for obtaining the desired parameter estimates.

Further discussion of the above estimation methods may be found in papers by Kao.[4, 11] Also, in a recent study by Weiss a method has been determined that may be used to estimate this parameter by transformed sample spacings.[12]

USE OF THE CONVERSION TABLES

One form of application that should be of considerable use is that of evaluating the quality protection afforded by a proposed or existing acceptance-sampling plan. A possibility of immediate interest is the use of a plan from the MIL-STD-105B Tables.

Example (1)

Suppose, for example, that a 105B plan with an Acceptable Quality Level (AQL) of 2.5% and Sample Size Letter J has been proposed for use. Reference to the 105B Tables shows that for single sampling a sample size of 75 items and an acceptance number of 4 is specified. Suppose life testing time is to be 80 hours with simply a count made of the test items failing by the end of that period. From inspection experience with the product to which the plan is to be applied, it seems most appropriate to assume a Weibull distribution with a value for β of 1 2/3. The lot size will be relatively large compared to the sample size of 75 so binomial probabilities for sample items can be assumed. Actually, Table III of MIL-STD-105B specifies that the lot size should be from 1300 to 3200, 501 to 800, and 181 to 300 for Inspection Levels I, II, and III respectively.

The first step is to determine the probability of acceptance, P(A), for various values of p'. These probabilities can readily be obtained from any one of the readily available tables of the cumulative binomial terms or tables of the incomplete beta distribution. They may also be read from the operating characteristic curves published as a part of the MIL-STD-105B Tables. A few of these values for this plan are shown in the first and second columns of the tabulation below. Next, the first of the conversion tables, Table 1, is used to obtain values for the ratio $(t/\mu) \times 100$ for each of the p' values. These table values are listed in the third column. Finally, using the value for t of 80 hours each of the $(t/\mu) \times 100$ ratios are converted to values for μ. For example, the ratio for a p' of 5% is 18.84. Thus $(80/\mu) \times 100 = 18.84$ or $\mu = 425$ hours. These computations have been made with results as shown in the last column. One may now note that if a lot is submitted to this plan whose mean life is 215 hours, the probability of its acceptance is only .01 or one in a hundred; on the other hand, if the mean life for a lot is 745 hours the probability of acceptance is .98. These probability and mean life figures based upon t = 80 hours can be plotted, if desired, to give the operating characteristic curve. (Of course similar OC curves for other known values of t may be plotted.) This curve is the one shown for $\beta = 1\ 2/3$ in Figure 3.

p' (in %)	P(A)	$(t/\mu) \times 100$	μ
2	.98	10.77	745
3	.92	13.78	580
4	.82	16.42	490
5	.68	18.84	425
6 1/2	.46	22.15	360
8	.27	25.20	315
10	.12	29.01	275
12	.04	32.58	245
15	.01	37.63	215

To indicate the importance of considering the shape of the life density for a product, operating characteristic curves for this plan have been computed and plotted in Figure 3 for other selected values for β. Included is a curve for the case in which β equals 1. This represents the exponential distribution widely used as a model in reliability and life-testing sampling inspection. From these curves it may be noted that if the underlying distribution is actually of the Weibull form and the exponential is assumed, the actual operating characteristics of the plan may differ very much from those contemplated. A discussion of the sensitivity of statistical procedures in current use to departures from the assumed exponentiality will be found in a paper by Zelen and Dannemiller.[13]

It may be noted in connection with this example that the MIL-STD-105B plans include matching double and multiple sampling plans. These offer alternative possibilities for reliability and life testing applications. If incoming lots are either quite good or quite bad (as is commonly the case), substantial reductions in the number of items that must be tested may be made. If items are expensive and if testing is destructive (as it most likely will be in life testing), a reduction in average sample size may be of importance. If the test period, t, is relatively long, however, the elapsed time required for testing a second sample (or subsequent ones in multiple sampling) when such samples are required to reach a decision may raise difficulties.

Example (2)

For a second example consider the case of a manufacturer who knows that his current production of a certain component has a mean life of approximately 52,000 cycles. Furthermore, he has learned from his past experience with life testing of these components that he can assume a β value of 1/2. A life test period of 1000 cycles seems justifiable and facilities are available for testing a sample of 150 items from each lot. This manufacturer would like to know what acceptance criteria to apply so that virtually all lots will be passed as long as the expected mean life of 52,000 cycles is maintained. He would also like to know what consumer protection will be afforded. A final question is whether for this application changing to a proposed test period of 300 cycles and a sample size of 500 items would yield comparable or better quality assurance.

The first step toward answers to these questions is to compute the $(t/\mu) \times 100$ ratio at the mean life considered acceptable. This ratio is $(1000/52{,}000) \times 100$ or 1.93. Entering Table 2 with this value gives (with rough interpolation) a value for p' of 18% for a β value of 1/2. Assuming a probability of acceptance of .95 is desired for lots at the acceptable quality level of 52,000 cycles for the lot mean, entering a table of the cumulative binomial distribution indicates an acceptance number, c, of 35 items gives this probability for a sample size of 150 items. This, then is the desired acceptance criteria.

A simple measure of consumer's protection is to find the lot mean value at which lots will likely be rejected. Suppose a probability of rejection of .90 (of acceptance of .10) seems to be a meaningful figure. Reference again to a binomial table indicates that for $n = 150$ and $c = 35$, the probability of rejection is .90 at a p' of approximately 28.4%. Entering Table 2 with this value gives a $(t/\mu) \times 100$ ratio of approximately 5.7. Substituting a value of 1000 cycles for t in this ratio and solving for μ gives a lot mean of 17,500 cycles. This figure for consumer's protection can be interpreted as follows--since this quality ($\mu = 17{,}500$) corresponds to a $P(A) = .10$, under this sampling plan ($n = 150$, $c = 35$) on the average 90% of the lots passed to the consumer will have a mean life of no less than 17,500 cycles. This may or may not represent adequate consumer protection. If it does not, a plan with a larger sample size must be designed and used.

An answer to the third question may be found by making similar computations for an n of 500 items and a value for t of 300 cycles. In this case $(t/\mu) \times 100$ equals $(300/52{,}000) \times 100$ or .58. From Table 2 it is found that p' is approximately 10% at this truncation ratio. Scanning a binomial table indicates an acceptance number of 62 will give a probability of acceptance of .95 or more when the sample size is 500 items. With this sample size and acceptance number, the probability of rejection is .90 at a p' value of approximately 14%. With this value for p', a $(t/\mu) \times 100$ value of approximately 1.15 is found from Table 1. Substituting 300 cycles for t in this ratio gives a lot mean value of 26,100 cycles as compared to 17,500 cycles for the first plan. Thus this combination of sample size and length of test period gives better discrimination between good and bad lots and the consumer is therefore better protected.

Example (3)

In this example, reference will be made to a case for which the component life can best be characterized by a mixture of two Weibul distributions. Kao[14] gives an example of this for the life of electron tubes. From the electron tube life experience, the wearout failures, i.e., drift of electrical properties beyond some set limits, invariably occur near the latter part of life. Hence the failures of electron tubes are classified both as of the wearout type and as of the non-wearout or catastrophic type, each type being represented by a sub-population of the whole. In electronic terms, these failure types are referred to as electrical rejects and inoperative rejects respectively. The catastrophic (or inoperative rejects) sub-population is assumed to start at time zero, i.e., the location parameter $\gamma_1 = 0$, when the components are exposed to risks. The wearout (or electrical rejects) sub-population is assumed not to start until some delayed period has elapsed, i.e., $\gamma_2 > 0$, since the limits set on the component drift depend on many factors such as environmental stress, maintenance policy, legal regulations, etc. Since, in general, failures due to wearout and non-wearout reasons are identifiable, it is possible to treat the two sub-populations separately.

Suppose that for some application of electron tubes, the manufacturer's past experience indicates that the Weibull shape parameter associated with the catastrophic sub-population, $\beta_1 = 1/2$ and that associated with the wearout sub-population $\beta_2 = 3\ 1/3$ are reasonable values, and furthermore that electrical drift or wearout failure has never been experienced prior to 1000 hours of life ($\gamma_1 = 0$, $\gamma_2 = 1000$). Suppose further that the manufacturer knows that approximately 2 1/2% of the total tube failures are of the inoperative type and that the mean life of his current production is,

$$\mu = (.025)(25{,}000) + (.975)(11{,}000) = 11{,}325 \text{ hrs}$$

where $\mu_1 = 25{,}000$ is the mean life of the catastrophic sub-population and $\mu_2 = 11{,}000$ is that of the wearout sub-population. (See the appendix of the paper by Kao[14] for the derivation of this formula.) A life test period of 500 hours for inoperatives and of 5000 hours for electrical drifts are recommended and acceptance numbers $c_1 = 2$ and $c_2 = 2$ for each failure type are considered satisfactory. What are the necessary sample sizes so that the producer's risk is no more than 5%? Also, what would be the consumer's protection under this sampling plan?

To answer these questions, the two sub-populations are treated separately and are denoted by subscripts 1 and 2 as done before for the inoperatives and electrical drifts respectively. For inoperatives, $(t_1/\mu_1) \times 100 = (500 \times 100)/25{,}000 = 2.0$. Entering Table 2 with this value gives a value for p_1' of 18.13% for a β value of 1/2. From a binomial table with $P(A) \geq .95$ and $p_1' = 18.13\%$, a value for $n_1 = 5$ is obtained. The same binomial table for $n_1 = 5$, $c_1 = 2$ and $P(A) \leq .10$ gives $p_1' = 75\%$. Entering Table 2 with this value gives $(t_1/\mu_1) \times 100 = 96.5$ and $\mu_1 = (500 \times 100)/96.5 = 518.24$ hours, a value which will be commented on later. For electrical drifts, γ_2 must be subtracted from t_2 and μ_2 giving new values for t_2 and μ_2 equal to 4,000 and 10,000 respectively. Hence, $(t_2/\mu_2) \times 100 = (4{,}000 \times 100)/10{,}000 = 40.0$. Entering Table 2 with this value gives for p_2' a figure of 3.25% for a β value of 3 1/3. From a binomial table with $P(A) \geq .95$ and $p_2' = 3.25\%$, it is found that $n_2 = 25$. The same binomial table for $n_2 = 25$, $c_2 = 2$, $P(A) \leq .10$ gives $p_2' = 20\%$. Entering Table 1 with this value gives $(t_2/\mu_2) \times 100 = 71.04$. Thus $\mu_2 = (4{,}000 \times 100)/71.04 = 5{,}631$ hours, which upon re-adding γ_2 gives 6,631 hours. Combining this value of corrected μ_2 with μ_1 obtained for inoperatives gives the consumer's protection expressed in terms of a mean value equal to,

$$\mu = (.025)(518.24) + (.975)(6{,}631) = 6{,}478 \text{ hours.}$$

This means under the sampling plan of running a life test for inoperatives of $n = 5$ and $c = 2$ for 500 hours and another life test for electrical drifts of $n = 25$ and $c = 2$ for 5000 hours, 90% of the lots passed to consumers will have a mean life of at least 6,478 hours. To illustrate the danger of extrapolation in a mixed distribution case, assume that the second test of 5000 hours duration was not run at all, then the producer could only base his conclusion upon the 500-hour test and claim as consumers' protection, with 90% confidence, a mean life of at least 518.24 hours, a result which is altogether too modest.

RELATIONSHIP BETWEEN ACCEPTABLE AND UNACCEPTABLE LOT QUALITY

Sampling plans are most conveniently cataloged, selected, or designed in terms of a producer's risk and a consumer's risk. Some lot quality figure will be specified as satisfactory and for lots of this quality or better the probability of acceptance should be high, conventionally .95 or more (the producer's risk of rejection small, .05 or less). For these plans for life testing, this specification will be a lot mean life, $\mu_{.95}$, at which $P(A) \geq .95$. Likewise, an unsatisfactory quality level will be specified for which the probability of acceptance will be low, conventionally .10. This specification will be a lot mean life, $\mu_{.10}$, at which $P(A) \leq .10$. (For other values of producer's and consumer's risks reference may be made to a paper by Kao.[15])

In plan selection or design, one objective is to find a combination of sample size and acceptance number which simultaneously yields the desired values for both the consumer risk and the producer risk. If working from tables of plans, the values for lot quality at the two risk figures may be listed. In the design of a plan, one may cut and try until a suitable plan is found in a manner suggested in Example 2. Also, factors are available which, in conjunction with the conversion tables supplied here, enable a direct determination to be made.[16]

A simple alternative solution for the form of plan discussed here is to make use of one of its properties, namely that for a given acceptance number, c, (and for a given value for β) the ratio between the lot means at the two risk values is approximately constant for all values of sample size, n. These ratios (or multipliers) have been determined for values for c ranging from 0 to 15 for each of the various values for β. They are presented in Table 3. The table values are in the form of multipliers for finding $\mu_{.95}$, given $\mu_{.10}$, or using the reciprocal of the multiplier, for finding $\mu_{.10}$, given $\mu_{.95}$. That is, $\mu_{.95}$ (for which $P(A) = .95$) is equal (approximately) to $\mu_{.10}$ (for which $P(A) = .10$) times the appropriate table multiplier. These multipliers may be used both to assist in evaluating the operating characteristics of some given plan and to assist in the design of a plan to meet some acceptance-inspection requirement.

Example (4)

For a certain purchased component the lot mean life should be at least 4,000 hours; this value is accordingly chosen for $\mu_{.10}$. Also, the producer has been informed that lots whose mean life is 10,000 hours or more are reasonably sure of acceptance through the sampling procedure. Accordingly, this value is to be used for $\mu_{.95}$. A value for β of 1 can be assumed. A testing period, t, of 200 hours has been specified. Values for sample size, n, and acceptance number, c, must be found to meet these requirements.

The ratio between the two lot means, $\mu_{.95}/\mu_{.10}$, is 10,000/4,000 or 2.5 . Examination of the table of mean life multipliers, Table 3, under the column for $\beta = 1$ indicates that an acceptance number, c, of 10 items will give this ratio. The $(t/\mu) \times 100$ ratio at $\mu_{.10}$ is $(200/4,000) \times 100$ or 5. Entering Table 2, the table of p', with this truncation ratio value of 5, gives a p' of 4.88%. Reference to a table of the cumulative binomial distribution or use of the Poisson approximation for c = 10 and p' = .0488 at P(A) = .10 shows that a sample size, n, of 315 items meets the requirements. A check for this solution can be made, if desired. For n = 315, c = 10, and P(A) = .95 the Poisson approximation indicates a p' of 1.96%. Entering Table 1, the table for per cent truncation, with this value for p', a value for $(t/\mu) \times 100$ of approximately 2.0 is found. Solving for $\mu_{.95}$ yields $(200/\mu) \times 100 = 2.0$ or $\mu_{.95} = 10,000$ which is the desired value.

TABLES OF SAMPLING PLANS

A set of tables of sampling inspection plans has been prepared, one table for each of the nine values for β for which the relationship between p' and $(t/\mu) \times 100$ has been established. These are presented as Tables 4a through 4i.

Each table lists values for the acceptance number, c, and for the minimum sample size, n, for a variety of objective t/μ ratios. The plans are designed so that if 100 times the ratio between the test time, t, and the mean life value for the lot, μ, or $(t/\mu) \times 100$ is equal to or greater than the selected column value in the table, the probability of acceptance, P(A), will be .10 or less. Stated otherwise, the plans assure with 90% confidence or more the acceptance of lots for which the $(t/\mu) \times 100$ ratio is equal to or less than the selected column or objective value. The ratios in the column headings (for which the plans have been designed) may thus be considered in the same way as lot tolerance per cent defective (LTPD) values are in describing operating characteristics of the widely used attribute and variables acceptance plans.

It has been assumed that in acceptance inspection for reliability the consumer's risk will be of primary concern. For this reason, these plans have been cataloged by P(A) = .10 ratios which measure consumer protection. However, in addition for each plan the $(t/\mu) \times 100$ ratio is given for which the probability of acceptance is .95 or more. Each such P(A) = .95 ratio value is shown in parentheses under the corresponding sample size number. These ratio values may be considered similar to acceptable quality level (AQL) values in indicating the producer's risk. If the mean life for the items in the lot is such that the t/μ ratio is equal to or less than the tabulated value, there is assurance with confidence of 95% or more that the lot will be accepted.

The two ratio values, one in the column heading and the other in parentheses below the sample size number, broadly describe the operating characteristics of each plan and so form a basis for making an appropriate choice for any acceptance inspection application. These values may also be used to determine approximately the operating characteristics of any acceptance plan that has been specified or that is in use and for which n and c match closely one of the table plans. It is easy to convert these ratios to hours, cycles or some other measure of lifelength to fit the product and test specifications involved. This will be illustrated by two examples which follow later.

In the preparation of these plans, binomial tables prepared by Grubbs[17] were employed for values for c up to 9 and for n up to 150. For higher values of c and for values for n up to 60 or so, the Pearson tables of the incomplete beta-function were used.[18] Higher values of n were determined by the Poisson approximation, using a table of np' values prepared by Cameron.[16] The Poisson match was checked and was found close, even for the smaller sample sizes and large values for p'. The slight differences that may exist in some cases is on the conservative side; the value for n is slightly larger than that theoretically required. As this is primarily an exploratory study, plans showing extremely large sample sizes have been included to indicate the order of magnitude involved and not with the expectation that samples of this size would ordinarily be used.

Example (5)

An acceptance inspection plan is required which will assure with 90% confidence a mean life for items of 4000 hours or more for each lot accepted. Also, it will be desirable to assure the producer that if the mean life for items in a lot is 25,000 hours or more, there will be a high probability (.95) of its acceptance.

A test period of 400 hours for the inspection of sample items has been specified. Through past experience it has been determined that the distribution of item life is of the Weibull form with β equal to approximately 1/2.

For these sampling plan specifications 100 times the ratio of test time, t, to mean life, $\mu_{.10}$, is (400/4,000) x 100 or 10 for which a probability of acceptance of .10 or less is desired. At the .95 probability value the ratio is (400/25,000) x 100 or 1.6. A plan approximating this may be found in Table 4b which gives plans for distributions for which $\beta = 1/2$. The column for which $(t/\mu) \times 100 = 10$ is entered and scanned for the ratio value 1.6 among the values listed in parentheses. This value is found well down in the column. The corresponding sampling inspection plan specifies a sample size, n, of 43 and an acceptance number, c, of 11.

Example (6)

A sampling inspection plan specifies that a random sample of 3000 items be drawn from the lot and tested for a period of 480 hours. If no more than 7 items fail before the end of the test period, the lot is to be accepted; if more than 7 items do not live through the test period, the lot is to be rejected. Life measurements for past inspection and research for the product to which the plan is to be applied indicate the distribution is of the Weibull form with β equal to approximately 1 2/3. The prospective user of this plan would like to know what quality protection will be given. Inspection of Table 4d which lists plans for $\beta = 1\ 2/3$ discloses a plan matching reasonably well the one specified, the plan for which c, the acceptance number, is 7 and n, the sample size, is 3,019. For this table plan the $(t/\mu) \times 100$ ratio at $P(A) = .10$ is 4. Substitution of the specified test period length of 480 hours for t gives $(480/\mu) \times 100 = 4$. Solving for μ gives 12,000 hours as the mean value for item life for the lot for which the probability of acceptance is .10 or less. A similar substitution for t using the ratio at which $P(A) = .95$ gives $(480/\mu) \times 100 = 2.1$. Solving for μ again gives 23,000 hours as a lot mean value for which the probability of acceptance is .95 or better. The values for the lot mean at these two probability values broadly, but very practically, describe the operating characteristics of the specified plan.

In the use of these tables of plans, several points of practice should be noted. First, in using the p' values associated with values for (t/μ) x100 for the Weibull distribution to find values for c and n, the binomial probability distribution has been used. Hence the size of the lot should be relatively large compared to the size of the sample for the stated probability values to precisely apply. Second, if a plan is not available for which a $(t/\mu) \times 100$ ratio in the column headings matches closely the desired ratio, to be conservative, a plan should be chosen from the column with the next smaller ratio heading. This will assure with confidence greater than 90% the specified mean life for acceptance. If the acceptable quality level (the ratio or mean life for which $P(A)=.95$) must also be guaranteed and a matching ratio value is not found in the selected column of plans, a plan with the next greater value should be selected. Lots equal to or better than the specified "acceptable quality" will have an assurance of greater than 95% of being accepted. With proper care, some rough interpolation may be employed between listed sample sizes (either down or across the table or both) to find a new plan having more nearly the desired characteristics. Finally, if a plan is found for which the desired and given ratios closely match but for practical reasons it seems desirable to round off the sample size to the nearest number ending in zero or five, such rounding off should be done to the next larger size. This will assure retention of the probability values of .10 or less for the ratios given in the column headings.

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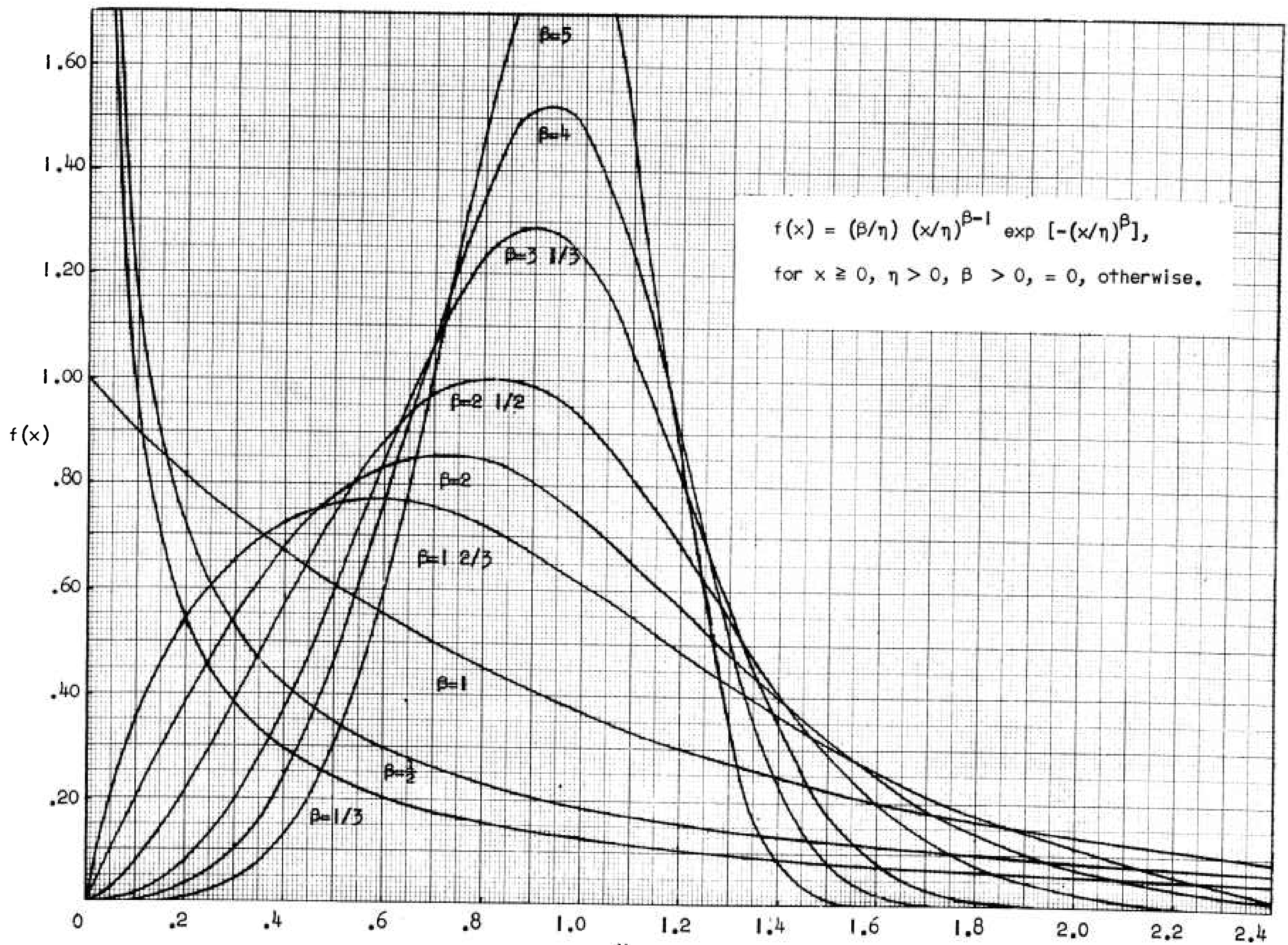

Figure 1. Plot of the Weibull Probability Density Function for Various Values of β - (η=1)

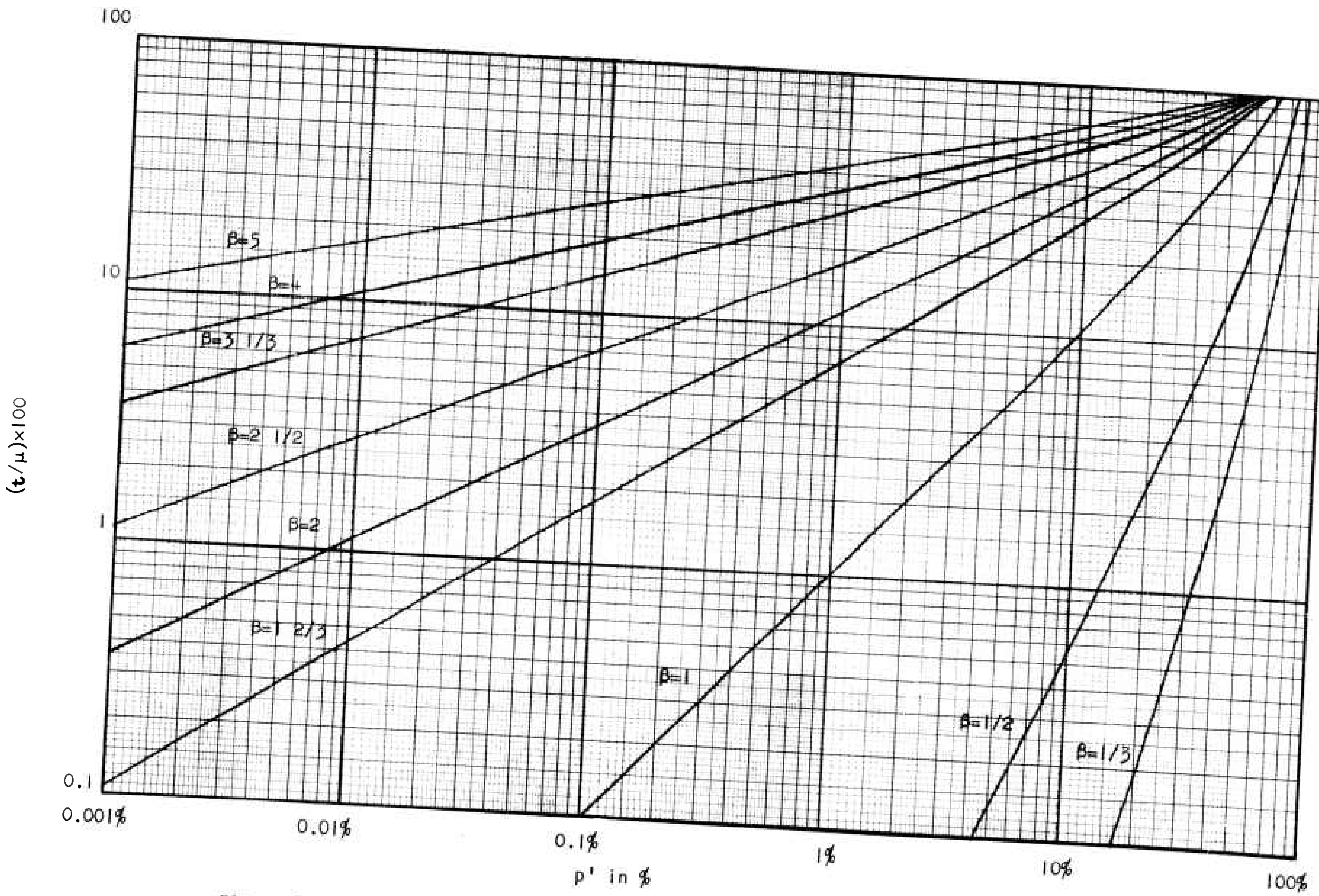

Figure 2. The Relationship Between $(t/\mu)\times 100$ and p' for Various Values of β

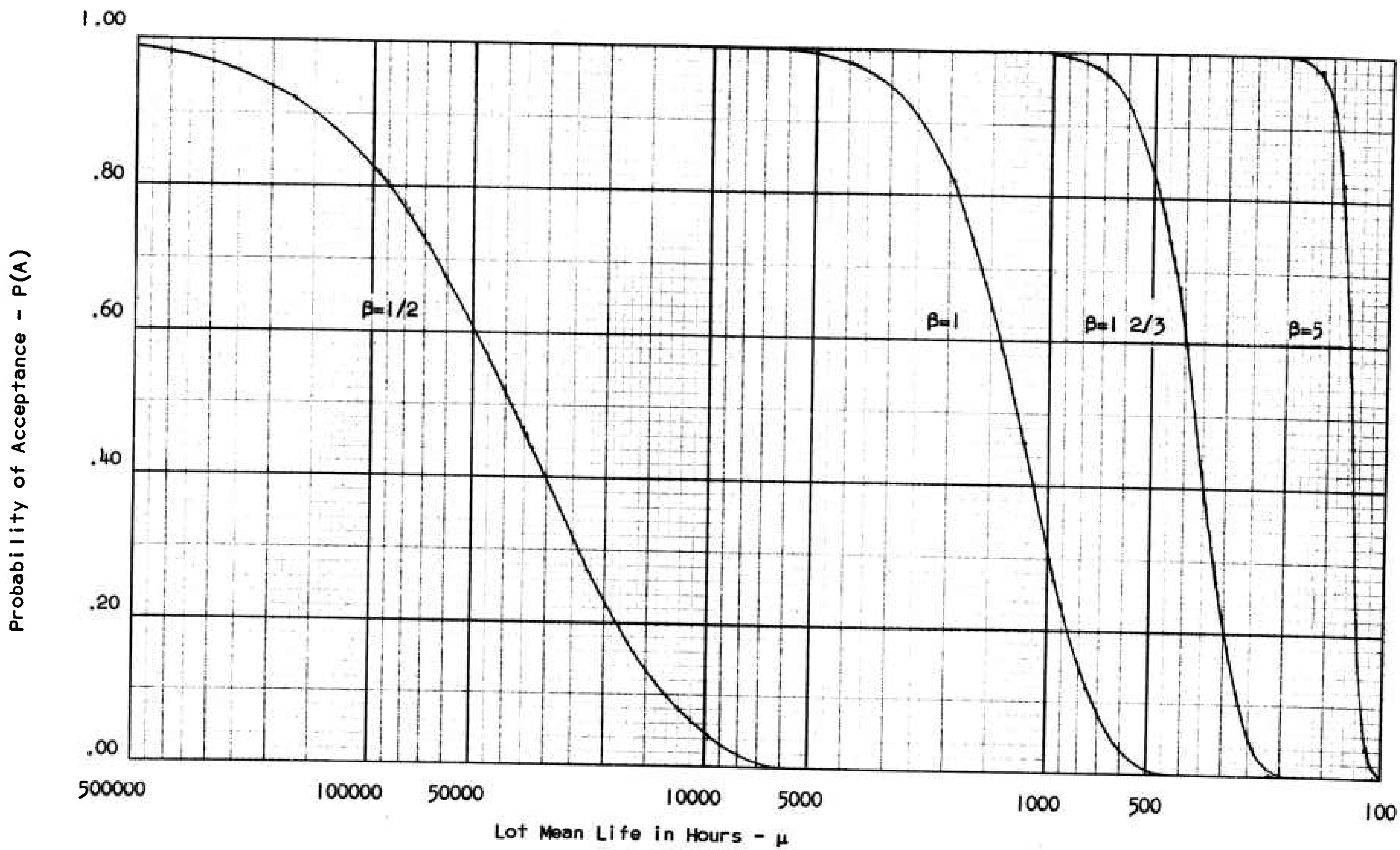

Figure 3. Operating Characteristic Curves for Various Values of β

$n = 75$ $c = 4$ $t = 80$ hrs.

TABLE 1

Table of Values for Per cent Truncation, $(t/\mu) \times 100$

p' (in %)	Shape Parameter - β								
	1/3	1/2	1	1 2/3	2	2 1/2	3 1/3	4	5
.010			.010	.45	1.13	2.83	7.03	11.03	17.26
.012			.012	.49	1.24	3.04	7.42	11.55	17.91
.015			.015	.57	1.38	3.32	7.94	12.21	18.72
.020			.020	.67	1.59	3.73	8.66	13.12	19.83
.025			.025	.77	1.78	4.08	9.26	13.87	20.74
.030			.030	.86	1.95	4.40	9.77	14.52	21.50
.040			.040	1.02	2.26	4.93	10.65	15.60	22.77
.050			.050	1.18	2.53	5.39	11.40	16.49	23.82
.065			.065	1.37	2.88	5.98	12.32	17.62	25.10
.080			.080	1.56	3.19	6.50	13.13	18.56	26.16
.100			.10	1.78	3.57	7.11	14.03	19.62	27.36
.12			.12	1.98	3.92	7.65	14.82	20.53	28.37
.15			.15	2.26	4.37	8.36	15.84	21.71	29.67
.20			.20	2.69	5.07	9.39	17.27	23.33	31.43
.25			.25	3.08	5.64	10.27	18.47	24.68	32.87
.30			.30	3.44	6.18	11.05	19.51	25.83	34.09
.40			.40	4.07	7.14	12.39	21.27	27.76	36.12
.50		.001	.50	4.67	7.99	13.55	22.75	29.36	37.76
.65		.002	.65	5.46	9.12	15.06	24.62	31.35	39.81
.80		.003	.80	6.19	10.11	16.36	26.21	33.03	41.50
1.00		.005	1.01	7.08	11.31	17.90	28.03	34.93	43.40
1.2		.007	1.21	7.90	12.40	19.26	29.62	36.57	45.02
1.5		.011	1.51	9.07	13.87	21.08	31.68	38.68	47.09
2.0		.020	2.02	10.77	16.03	23.67	34.56	41.59	49.90
2.5		.032	2.53	12.33	17.95	25.90	36.98	44.01	52.21
3.0		.047	3.05	13.78	19.69	27.89	39.09	46.09	54.17
4.0	.001	.083	4.08	16.42	22.79	31.35	42.69	49.59	57.45
5.0	.002	.13	5.13	18.84	25.58	34.35	45.71	52.50	60.13
6.5	.005	.23	6.72	22.15	29.25	38.28	49.57	56.18	63.46
8.0	.010	.35	8.34	25.20	32.59	41.72	52.88	59.29	66.26
10.0	.020	.56	10.54	29.01	36.63	45.82	56.73	62.85	69.44
12	.034	.82	12.78	32.58	40.34	49.50	60.11	65.96	72.18
15	.070	1.32	16.25	37.63	45.48	54.49	64.60	70.05	75.73
20	.18	2.49	22.31	45.51	53.30	61.85	71.04	75.83	80.68
25	.40	4.14	28.77	52.99	60.53	68.47	76.67	80.80	84.89
30	.76	6.36	35.37	60.29	67.39	74.62	81.79	85.26	88.62
40	2.22	13.04	51.08	74.79	80.64	86.15	91.09	93.27	95.22
50	5.55	24.02	69.31	89.82	93.95	97.33	99.82	100.67	101.21
65	19.28	55.10	104.98	115.23	115.61	114.92	113.06	111.68	109.98
80	69.48	129.52	160.94	148.91	143.14	136.34	128.53	124.27	119.79

TABLE 2

Table of Probability Values at Truncation Point, p' (%)

(t/μ) x 100	Shape Parameter - β								
	1/3	1/2	1	1 2/3	2	2 1/2	3 1/3	4	5
.010	8.09	1.40	.010						
.012	8.57	1.54	.012						
.015	9.20	1.72	.015						
.020	10.08	1.98	.020						
.025	10.82	2.21	.025						
.030	11.45	2.42	.030						
.040	12.53	2.79	.040						
.050	13.43	3.11	.050						
.065	14.56	3.54	.065						
.080	15.52	3.92	.080						
.100	16.61	4.37	.10						
.12	17.56	4.78	.12	.001					
.15	18.78	5.33	.15	.002					
.20	20.46	6.13	.20	.003					
.25	21.86	6.83	.25	.004					
.30	23.06	7.45	.30	.005					
.40	25.06	8.56	.40	.009	.001				
.50	26.71	9.52	.50	.012	.002				
.65	28.76	10.78	.65	.019	.003				
.80	30.47	11.88	.80	.027	.005				
1.00	32.40	13.19	1.00	.038	.008				
1.2	34.03	14.35	1.19	.052	.011	.001			
1.5	36.12	15.90	1.49	.076	.018	.002			
2.0	38.94	18.13	1.98	.12	.031	.004			
2.5	41.22	20.04	2.47	.18	.049	.007			
3.0	43.14	21.73	2.96	.24	.071	.012	.001		
4.0	46.28	24.64	3.92	.39	.13	.024	.002		
5.0	48.80	27.11	4.88	.56	.20	.041	.003		
6.5	50.90	30.27	6.29	.89	.33	.080	.008	.001	
8.0	54.30	32.97	7.69	1.22	.50	.13	.015	.003	
10.0	56.98	36.06	9.52	1.77	.78	.23	.033	.007	.001
12	59.19	38.73	11.31	2.39	1.12	.37	.060	.014	.002
15	61.92	42.17	13.93	3.45	1.75	.64	.13	.034	.005
20	65.45	46.87	18.13	5.51	3.09	1.32	.33	.11	.021
25	68.17	50.69	22.12	7.89	4.79	2.29	.69	.26	.064
30	70.37	53.91	25.92	10.56	6.82	3.59	1.26	.55	.16
40	73.79	59.12	32.97	16.47	11.81	7.23	3.25	1.71	.67
50	76.36	63.21	39.35	22.98	17.83	11.28	6.72	4.13	2.02
65	79.28	68.02	47.80	33.26	28.24	22.32	15.37	11.35	7.29
80	81.49	71.77	55.07	43.54	39.51	34.59	28.35	24.15	19.25
100	83.75	75.69	63.21	56.35	54.41	52.36	50.41	49.08	47.93

TABLE 3

Table of Mean Life Multipliers

Approximate Values for $\mu_{.95}/\mu_{.10}$

c	β								
	1/3	1/2	1	1 2/3	2	2 1/2	3 1/3	4	5
0			45	10	6.7	4.6	3.1	2.6	2.2
1	2000	150	11	4.3	3.3	2.6	2.1	1.8	1.6
2	325	45	6.7	3.1	2.6	2.1	1.8	1.6	1.5
3	140	25	5.0	2.6	2.2	1.9	1.6	1.5	1.4
4	75	17	4.1	2.3	2.0	1.8	1.5	1.4	1.3
5	50	13	3.6	2.2	1.9	1.7	1.5	1.4	1.3
6	35	11	3.2	2.0	1.8	1.6	1.4	1.3	1.3
7	27	9.1	3.0	1.9	1.7	1.6	1.4	1.3	1.3
8	23	8.0	2.8	1.9	1.7	1.5	1.4	1.3	1.2
9	20	7.0	2.7	1.8	1.6	1.5	1.3	1.3	1.2
10	18	6.4	2.5	1.8	1.6	1.5	1.3	1.3	1.2
11	16	6.0	2.4	1.7	1.6	1.4	1.3	1.3	1.2
12	14	5.6	2.3	1.7	1.5	1.4	1.3	1.2	1.2
13	13	5.2	2.2	1.6	1.5	1.4	1.3	1.2	1.2
14	12	5.0	2.2	1.6	1.5	1.4	1.3	1.2	1.2
15	11	4.8	2.1	1.6	1.5	1.4	1.3	1.2	1.2

TABLE 4a

Table of Sampling Plans for $\beta = 1/3$

c	n												
	$(t/\mu) \times 100$ Ratio for which $P(A) = .10$ (or Less)												
	100	50	25	10	5	2.5	1	0.5	0.25	0.1	0.05	0.025	0.010
0	1	2	2	3	4	5	6	8	10	13	16	21	28
1	3 (.05)	4 (.02)	4 (.01)	6	7	8	11	14	17	22	28	35	47
2	5 (.15)	5 (.16)	6 (.08)	8 (.03)	9 (.02)	12 (.01)	15	19	23	31	38	48	65
3	6 (.53)	7 (.27)	8 (.17)	10 (.07)	12 (.04)	15 (.02)	19 (.01)	24	29	39	48	60	81
4	7 (1.2)	8 (.66)	10 (.33)	12 (.14)	15 (.06)	18 (.03)	23 (.01)	28 (.01)	35	46	58	72	97
5	9 (1.3)	10 (.80)	11 (.54)	14 (.20)	17 (.10)	21 (.05)	27 (.02)	33 (.01)	41 (.01)	54	67	84	113
6	10 (2.2)	11 (1.2)	13 (.65)	16 (.28)	19 (.15)	24 (.07)	31 (.03)	37 (.02)	46 (.01)	61	76	95	128
7	11 (2.6)	13 (1.4)	15 (.76)	18 (.35)	22 (.18)	26 (.10)	34 (.04)	42 (.02)	52 (.01)	69	86	107	143
8	13 (2.9)	14 (1.7)	16 (1.1)	20 (.44)	24 (.23)	29 (.12)	38 (.05)	47 (.02)	58 (.01)	76	95	118	161
9	14 (3.8)	16 (2.1)	18 (1.2)	22 (.52)	27 (.24)	32 (.13)	41 (.06)	51 (.03)	63 (.01)	83 (.01)	103	129	176
10	15 (4.6)	17 (2.5)	20 (1.2)	24 (.58)	29 (.28)	35 (.14)	45 (.06)	55 (.03)	71 (.01)	93 (.01)	115	143	191
11	16 (5.0)	19 (2.8)	21 (1.7)	26 (.68)	31 (.34)	38 (.16)	48 (.07)	60 (.03)	76 (.02)	100 (.01)	124	154	206
12	18 (5.5)	20 (3.1)	23 (1.7)	28 (.72)	34 (.34)	41 (.16)	52 (.07)	67 (.03)	82 (.02)	108 (.01)	133	165	220
13	19 (6.2)	21 (3.6)	24 (1.9)	30 (.76)	36 (.40)	43 (.18)	56 (.07)	71 (.04)	87 (.02)	115 (.01)	142	176	235
14	20 (6.7)	23 (3.9)	26 (2.1)	32 (.85)	38 (.45)	46 (.22)	60 (.09)	76 (.04)	93 (.02)	122 (.01)	150	187	249
15	22 (7.0)	24 (4.2)	28 (2.2)	34 (.95)	40 (.45)	49 (.22)	63 (.09)	80 (.04)	98 (.02)	129 (.01)	159	197	264

$(t/\mu) \times 100$ ratios in parentheses are for $P(A) = .95$ (or more)

TABLE 4b

Table of Sampling Plans for $\beta = 1/2$

c	n												
	$(t/\mu) \times 100$ Ratio for which P(A) = .10 (or less)												
	100	50	25	10	5	2.5	1	0.5	0.25	0.1	0.05	0.025	0.01
0	2 (.03)	3 (.02)	4 (.01)	6	8	11	17	23	33	52	73	103	165
1	4 (.52)	5 (.32)	7 (.16)	10 (.07)	13 (.04)	18 (.02)	28 (.01)	40	56	88	124	177	278
2	5 (2.2)	7 (.94)	9 (.54)	13 (.24)	18 (.12)	25 (.05)	39 (.02)	55 (.01)	77 (.01)	120	172	241	381
3	7 (3.3)	9 (1.7)	12 (.85)	17 (.40)	23 (.21)	32 (.10)	49 (.04)	69 (.02)	96 (.01)	153	215	303	478
4	9 (4.1)	11 (2.5)	14 (1.4)	20 (.60)	28 (.30)	38 (.15)	59 (.06)	82 (.03)	115 (.02)	183 (.01)	258	362	571
5	10 (6.4)	13 (3.3)	16 (1.9)	24 (.75)	32 (.40)	45 (.19)	68 (.08)	96 (.04)	134 (.02)	213 (.01)	299	420	663
6	12 (7.2)	14 (4.7)	19 (2.2)	27 (.94)	37 (.46)	51 (.24)	78 (.10)	109 (.05)	155 (.02)	242 (.01)	339	477	753
7	13 (9.7)	16 (5.3)	21 (2.6)	30 (1.1)	41 (.55)	57 (.28)	87 (.11)	122 (.06)	173 (.03)	270 (.01)	379 (.01)	533	841
8	14 (12)	18 (6.0)	23 (3.2)	34 (1.2)	46 (.61)	63 (.33)	96 (.13)	134 (.07)	191 (.03)	298 (.01)	418 (.01)	589	929
9	16 (12)	20 (6.4)	26 (3.3)	37 (1.4)	50 (.73)	69 (.36)	105 (.14)	147 (.08)	208 (.04)	326 (.01)	457 (.01)	643	1,015
10	17 (15)	22 (6.8)	28 (3.7)	40 (1.5)	54 (.78)	77 (.38)	118 (.15)	162 (.08)	226 (.04)	353 (.02)	496 (.01)	698	1,101
11	19 (16)	23 (8.0)	30 (4.1)	43 (1.6)	58 (.83)	83 (.40)	126 (.16)	175 (.08)	244 (.04)	380 (.02)	534 (.01)	752	1,186
12	20 (17)	25 (8.8)	32 (4.6)	47 (1.7)	66 (.87)	89 (.42)	135 (.17)	187 (.09)	261 (.04)	407 (.02)	572 (.01)	805	1,271
13	22 (18)	27 (9.0)	34 (5.0)	50 (1.9)	70 (.90)	95 (.44)	144 (.19)	200 (.09)	278 (.05)	434 (.02)	610 (.01)	858	1,355
14	23 (19)	29 (9.2)	37 (5.0)	53 (2.0)	75 (.92)	101 (.47)	153 (.20)	212 (.10)	295 (.05)	461 (.02)	648 (.01)	911 (.01)	1,438
15	25 (19)	31 (9.4)	39 (5.0)	56 (2.2)	79 (.98)	107 (.49)	162 (.21)	224 (.11)	312 (.05)	488 (.02)	685 (.01)	964 (.01)	1,521

$(t/\mu) \times 100$ ratios in parentheses are for P(A) = .95 (or more)

TABLE 4c

Table of Sampling Plans for $\beta = 1$

c	n												
	$(t/\mu) \times 100$ Ratio for which P(A) = .10 (or less)												
	100	50	25	10	5	2.5	1	0.5	0.25	0.1	0.05	0.025	0.01
0	3 (1.7)	5 (1.0)	10 (.51)	24 (.20)	46 (.11)	92 (.06)	231 (.02)	461 (.01)	922	2,303	4,606	9,212	230-2
1	5 (8.0)	9 (4.2)	17 (2.1)	40 (.90)	79 (.45)	158 (.22)	389 (.09)	778 (.05)	1,556 (.02)	3,890 (.01)	7,780	156-2	389-2
2	7 (14)	12 (7.4)	23 (3.7)	55 (1.5)	108 (.76)	216 (.38)	533 (.15)	1,065 (.08)	2,129 (.04)	5,322 (.02)	106-2 (.01)	213-2	532-2
3	9 (19)	15 (10)	29 (5.0)	69 (2.0)	135 (1.0)	271 (.50)	669 (.20)	1,337 (.10)	2,673 (.05)	6,681 (.02)	134-2 (.01)	267-2	668-2
4	11 (22)	19 (12)	34 (6.2)	82 (2.4)	164 (1.2)	324 (.61)	800 (.24)	1,600 (.12)	3,200 (.06)	8,000 (.02)	160-2 (.01)	320-2	800-2
5	13 (25)	22 (13)	40 (7.0)	96 (2.8)	191 (1.4)	376 (.69)	928 (.28)	1,855 (.14)	3,710 (.07)	9,275 (.03)	186-2 (.01	371-2 (.01)	928-2
6	14 (30)	25 (15)	46 (7.5)	109 (3.0)	216 (1.5)	427 (.77)	1,054 (.31)	2,107 (.16)	4,213 (.08)	105-2 (.03)	211-2 (.02)	421-2 (.01)	105-3
7	16 (33)	28 (16)	51 (8.2)	122 (3.3)	242 (1.7)	477 (.83)	1,178 (.34)	2,355 (.17)	4,709 (.08)	118-2 (.03)	235-2 (.02)	471-2 (.01)	118-3
8	18 (35)	31 (17)	57 (9.0)	135 (3.5)	267 (1.8)	527 (.89)	1,300 (.36)	2,600 (.18)	5,200 (.09)	130-2 (.04)	260-2 (.02)	520-2 (.01)	130-3
9	20 (36)	34 (18)	62 (9.3)	147 (3.7)	292 (1.9)	576 (.94)	1,421 (.38)	2,842 (.19)	5,683 (.10)	142-2 (.04)	284-2 (.02)	568-2 (.01)	142-3
10	22 (38)	37 (19)	70 (9.5)	162 (3.9)	316 (2.0)	624 (1.0)	1,541 (.40)	3,082 (.20)	6,163 (.10)	154-2 (.04)	308-2 (.02)	616-2 (.01)	154-3
11	23 (40)	40 (20)	76 (9.8)	175 (4.0)	341 (2.1)	672 (1.1)	1,660 (.42)	3,320 (.21)	6,640 (.10)	166-2 (.04)	332-2 (.02)	664-2 (.01)	166-3
12	25 (42)	43 (20)	81 (10)	187 (4.2)	365 (2.2)	720 (1.1)	1,780 (.43)	3,557 (.22)	7,113 (.11)	178-2 (.04)	336-2 (.02)	711-2 (.01)	178-3
13	27 (42)	45 (21)	86 (10)	200 (4.3)	389 (2.2)	768 (1.1)	1,896 (.45)	3,792 (.22)	7,584 (.11)	190-2 (.05)	379-2 (.02)	758-2 (.01)	190-3
14	29 (44)	48 (22)	91 (11)	212 (4.4)	413 (2.3)	815 (1.1)	2,013 (.46)	4,026 (.23)	8,052 (.12)	201-2 (.05)	403-2 (.02)	805-2 (.01)	201-3
15	31 (44)	51 (22)	97 (11)	224 (4.6)	437 (2.4)	863 (1.2)	2,130 (.47)	4,260 (.24)	8,517 (.12)	213-2 (.05)	426-2 (.02)	852-2 (.01)	213-3

$(t/\mu) \times 100$ ratios in parentheses are for P(A) = .95 (or more)

The figure following the dash in sample size numbers shows the number of zeros to add; for example, 154-3 = 154,000.

TABLE 4d

Table of Sampling Plans for $\beta = 1\ 2/3$

c	n												
	$(t/\mu) \times 100$ Ratio for which $P(A) = .10$ (or less)												
	100	50	25	15	10	8	5	4	2.5	1.5	1	0.5	0.25
0	3 (9.8)	9 (5.0)	28 (2.5)	66 (1.5)	129 (1.0)	189 (.80)	412 (.50)	591 (.40)	1,280 (.25)	3,031 (.15)	6,061 (.10)	184-2 (.05)	576-2 (.03)
1	6 (22)	16 (12)	48 (5.9)	112 (3.5)	220 (2.3)	319 (1.9)	695 (1.2)	998 (.96)	2,162 (.60)	5,119 (.35)	102-2 (.23)	311-2 (.11)	973-2 (.06)
2	8 (31)	22 (16)	66 (8.1)	155 (4.8)	301 (3.2)	437 (2.6)	951 (1.6)	1,365 (1.3)	2,957 (.81)	7,003 (.49)	140-2 (.32)	426-2 (.16)	133-3 (.08)
3	10 (38)	28 (19)	83 (9.6)	194 (5.7)	378 (3.8)	548 (3.1)	1,194 (1.9)	1,714 (1.5)	3,712 (.96)	8,791 (.58)	176-2 (.38)	534-2 (.19)	167-3 (.09)
4	12 (42)	33 (21)	100 (11)	232 (6.4)	452 (4.3)	656 (3.4)	1,428 (2.1)	2,050 (1.7)	4,442 (1.1)	105-2 (.65)	210-2 (.42)	640-2 (.21)	200-3 (.10)
5	14 (46)	39 (23)	116 (12)	269 (6.9)	525 (4.6)	761 (3.7)	1,657 (2.3)	2,379 (1.9)	5,153 (1.2)	122-2 (.69)	244-2 (.45)	742-2 (.23)	232-3 (.11)
6	17 (47)	44 (25)	132 (12)	306 (7.4)	596 (4.9)	864 (3.9)	1,881 (2.5)	2,701 (2.0)	5,852 (1.3)	139-2 (.75)	277-2 (.48)	843-2 (.25)	263-3 (.12)
7	19 (50)	49 (26)	148 (13)	342 (7.7)	666 (5.2)	965 (4.2)	2,102 (2.6)	3,019 (2.1)	6,540 (1.3)	155-2 (.78)	310-2 (.51)	942-2 (.26)	294-3 (.13)
8	21 (52)	54 (27)	165 (13)	377 (8.1)	735 (5.4)	1,066 (4.3)	2,321 (2.7)	3,333 (2.2)	7,220 (1.4)	171-2 (.80)	342-2 (.53)	104-3 (.27)	325-3 (.13)
9	23 (54)	59 (28)	181 (14)	412 (8.3)	803 (5.6)	1,165 (4.5)	2,537 (2.8)	3,643 (2.2)	7,893 (1.4)	187-2 (.84)	374-2 (.55)	114-3 (.28)	355-3 (.14)
10	24 (57)	68 (28)	196 (14)	447 (8.6)	871 (5.7)	1,263 (4.6)	2,752 (2.9)	3,951 (2.3)	8,560 (1.5)	203-2 (.86)	405-2 (.57)	123-3 (.29)	385-3 (.14)
11	26 (59)	73 (29)	211 (15)	482 (8.9)	938 (5.9)	1,361 (4.7)	2,964 (2.9)	4,256 (2.3)	9,222 (1.5)	218-2 (.89)	437-2 (.58)	133-3 (.30)	415-3 (.15)
12	28 (60)	78 (29)	226 (15)	516 (9.0)	1,005 (6.0)	1,458 (4.8)	3,176 (3.0)	4,560 (2.4)	9,879 (1.5)	234-2 (.90)	468-2 (.59)	142-3 (.30)	445-3 (.15)
13	30 (61)	83 (29)	241 (15)	550 (9.1)	1,072 (6.1)	1,554 (4.9)	3,386 (3.1)	4,862 (2.5)	105-2 (1.6)	249-2 (.92)	499-2 (.61)	152-3 (.31)	475-3 (.15)
14	33 (61)	88 (30)	256 (15)	584 (9.3)	1,138 (6.3)	1,650 (5.0)	3,595 (3.1)	5,162 (2.5)	112-2 (1.6)	265-2 (.94)	530-2 (.62)	161-3 (.31)	505-3 (.16)
15	35 (62)	93 (30)	270 (16)	618 (9.4)	1,203 (6.4)	1,746 (5.1)	3,803 (3.2)	5,460 (2.6)	118-2 (.16)	280-2 (.95)	560-2 (.63)	170-3 (.32)	535-3 (.16)

$(t/\mu) \times 100$ ratios in parentheses are for $P(A) = .95$ (or more)

The figure following the dash in sample size numbers shows the number of zeros to add; for example, 218-2 = 21,800.

TABLE 4e

Table of Sampling Plans for β = 2

c	n — (t/μ) x 100 Ratio for which P(A) = .10 (or less)												
	100	50	25	15	12	10	8	5	4	2.5	1.5	1	0.5
0	3 (15)	12 (7.4)	47 (3.7)	131 (2.2)	206 (1.8)	296 (1.5)	461 (1.2)	1,152 (.76)	1,772 (.62)	4,700 (.38)	128-2 (.23)	288-2 (.16)	115-3 (.08)
1	6 (29)	21 (15)	80 (7.5)	223 (4.5)	348 (3.6)	499 (3.0)	778 (2.4)	1,945 (1.5)	2,993 (1.2)	7,939 (.76)	216-2 (.46)	486-2 (.30)	194-3 (.15)
2	8 (38)	29 (19)	110 (9.8)	305 (5.8)	476 (4.7)	683 (3.9)	1,064 (3.1)	2,661 (2.0)	4,094 (1.6)	109-2 (.98)	296-2 (.60)	665-2 (.40)	266-3 (.20)
3	11 (43)	36 (22)	138 (11)	382 (6.7)	597 (5.4)	857 (4.5)	1,336 (3.6)	3,341 (2.3)	5,140 (1.8)	136-2 (1.1)	371-2 (.70)	835-2 (.47)	334-3 (.24)
4	13 (48)	43 (24)	167 (12)	457 (7.4)	714 (5.9)	1,025 (5.0)	1,599 (4.0)	3,977 (2.5)	6,150 (2.0)	163-2 (1.2)	444-2 (.76)	999-2 (.51)	400-3 (.26)
5	15 (52)	50 (26)	194 (13)	530 (7.9)	829 (6.3)	1,190 (5.3)	1,855 (4.2)	4,638 (2.7)	7,135 (2.2)	189-2 (1.3)	515-2 (.81)	116-3 (.54)	464-3 (.28)
6	17 (55)	57 (28)	220 (14)	602 (8.3)	941 (6.6)	1,351 (5.6)	2,106 (4.5)	5,214 (2.8)	8,102 (2.3)	215-2 (1.4)	585-2 (.85)	132-3 (.57)	527-3 (.29)
7	19 (58)	64 (29)	245 (14)	673 (8.7)	1,051 (6.9)	1,510 (5.8)	2,354 (4.6)	5,886 (2.9)	9,055 (2.4)	240-2 (1.5)	654-2 (.88)	147-3 (.60)	589-3 (.30)
8	21 (60)	71 (30)	272 (15)	743 (9.0)	1,161 (7.2)	1,667 (6.0)	2,600 (4.8)	6,498 (3.0)	9,997 (2.4)	265-2 (1.5)	722-2 (.91)	162-3 (.62)	650-3 (.31)
9	23 (62)	77 (30)	297 (15)	812 (9.2)	1,274 (7.4)	1,822 (6.1)	2,841 (4.9)	7,103 (3.1)	109-2 (2.5)	290-2 (1.5)	789-2 (.94)	178-3 (.63)	710-3 (.32)
10	26 (62)	87 (31)	322 (15)	881 (9.4)	1,376 (7.5)	1,976 (6.3)	3,081 (5.0)	7,704 (3.2)	119-2 (2.6)	314-2 (1.6)	856-2 (.96)	193-3 (.65)	770-3 (.33)
11	28 (63)	94 (31)	347 (16)	949 (9.6)	1,482 (7.7)	2,128 (6.4)	3,320 (5.1)	8,299 (3.3)	128-2 (2.6)	339-2 (1.6)	922-2 (9.8)	207-3 (.66)	830-3 (.34)
12	30 (64)	100 (32)	372 (16)	1,017 (9.8)	1,588 (7.8)	2,280 (6.5)	3,556 (5.2)	8,891 (3.3)	137-2 (2.7)	363-2 (1.6)	988-2 (1.0)	222-3 (.67)	889-3 (.34)
13	32 (65)	107 (32)	396 (16)	1,084 (10)	1,693 (8.0)	2,431 (6.6)	3,792 (5.3)	9,479 (3.4)	146-2 (2.7)	387-2 (1.7)	105-3 (1.0)	237-3 (.68)	948-3 (.35)
14	34 (66)	113 (33)	421 (17)	1,151 (10)	1,798 (8.1)	2,581 (6.7)	4,026 (5.4)	101-2 (3.4)	155-2 (2.8)	411-2 (1.7)	112-3 (1.0)	252-3 (.69)	101-4 (.35)
15	36 (67)	120 (33)	445 (17)	1,217 (10)	1,902 (8.2)	2,730 (6.8)	4,258 (5.5)	106-2 (3.5)	164-2 (2.8)	435-2 (1.7)	118-3 (1.0)	266-3 (.70)	106-4 (.36)

(t/μ) x 100 ratios in parentheses are for P(A) = .95 (or more)

The figure following the dash in sample size numbers shows the number of zeros to add; for example, 118-3 = 118,000.

TABLE 4f

Table of Sampling Plans for $\beta = 2\ 1/2$

c	n												
	$(t/\mu) \times 100$ Ratio for which P(A) = .10 (or less)												
	100	50	40	25	15	12	10	8	6.5	5	4	2.5	1.5
0	4 (20)	20 (10)	31 (8.7)	100 (5.4)	360 (3.3)	623 (2.6)	1,002 (2.2)	1,772 (1.7)	2,879 (1.4)	5,618 (1.1)	9,596 (.88)	320-2 (.55)	115-3 (.33)
1	6 (37)	34 (18)	53 (15)	170 (9.6)	608 (5.7)	1,052 (4.6)	1,692 (3.8)	2,993 (3.0)	4,863 (2.5)	9,488 (1.9)	162-2 (1.5)	540-2 (.96)	194-3 (.58)
2	9 (45)	46 (23)	72 (19)	233 (12)	832 (7.0)	1,439 (5.6)	2,314 (4.7)	4,094 (3.7)	6,653 (3.1)	130-2 (2.3)	222-2 (1.9)	739-2 (1.2)	266-3 (.71)
3	11 (51)	58 (25)	91 (21)	292 (13)	1,044 (7.8)	1,806 (6.3)	2,905 (5.2)	5,140 (4.2)	8,352 (3.4)	163-2 (2.6)	278-2 (2.1)	928-2 (1.3)	334-3 (.80)
4	13 (56)	70 (27)	109 (23)	350 (14)	1,250 (8.5)	2,161 (6.8)	3,476 (5.6)	6,150 (4.5)	9,993 (3.7)	195-2 (2.8)	333-2 (2.3)	111-3 (1.4)	400-3 (.86)
5	16 (58)	81 (29)	127 (24)	406 (15)	1,450 (9.0)	2,507 (7.2)	4,033 (6.0)	7,135 (4.7)	116-2 (3.9)	226-2 (3.0)	386-2 (2.4)	129-3 (1.5)	464-3 (.91)
6	18 (61)	92 (30)	144 (25)	460 (16)	1,646 (9.4)	2,847 (7.5)	4,580 (6.2)	8,102 (4.9)	132-2 (4.1)	257-2 (3.1)	439-2 (2.5)	146-3 (1.6)	527-3 (.94)
7	20 (64)	103 (31)	163 (25)	515 (16)	1,840 (9.7)	3,182 (7.7)	5,118 (6.4)	9,055 (5.1)	147-2 (4.2)	287-2 (3.2)	490-2 (2.6)	163-3 (1.6)	589-3 (.97)
8	22 (66)	113 (32)	180 (26)	568 (17)	2,031 (10)	3,513 (8.0)	5,650 (6.6)	9,997 (5.2)	162-2 (4.3)	317-2 (3.3)	541-2 (2.7)	180-3 (1.6)	650-3 (1.0)
9	25 (67)	124 (32)	197 (27)	621 (17)	2,220 (10)	3,840 (8.1)	6,177 (6.7)	109-2 (5.3)	178-2 (4.4)	346-2 (3.4)	592-2 (2.7)	197-3 (1.7)	710-3 (1.0)
10	27 (68)	137 (33)	214 (27)	673 (17)	2,408 (10)	4,164 (8.3)	6,699 (6.8)	119-2 (5.4)	193-2 (4.5)	376-2 (3.4)	642-2 (2.8)	214-3 (1.7)	770-3 (1.0)
11	29 (69)	148 (33)	230 (28)	725 (18)	2,594 (11)	4,486 (8.4)	7,217 (7.0)	128-2 (5.5)	207-2 (4.6)	405-2 (3.5)	692-2 (2.8)	231-3 (1.7)	830-3 (1.1)
12	31 (70)	158 (34)	246 (28)	777 (18)	2,779 (11)	4,806 (8.6)	7,732 (7.1)	137-2 (5.6)	222-2 (4.6)	434-2 (3.5)	741-2 (2.9)	247-3 (1.8)	889-3 (1.1)
13	33 (71)	168 (34)	263 (29)	828 (18)	2,963 (11)	5,124 (8.7)	8,243 (7.1)	146-2 (5.7)	237-2 (4.7)	462-2 (3.6)	790-2 (2.9)	263-3 (1.8)	948-3 (1.1)
14	35 (72)	179 (35)	279 (29)	879 (18)	3,145 (11)	5,440 (8.8)	8,752 (7.2)	155-2 (5.8)	252-2 (4.7)	491-2 (3.6)	839-2 (2.9)	280-3 (1.8)	101-4 (1.1)
15	37 (73)	189 (36)	295 (29)	930 (19)	3,327 (11)	5,755 (8.8)	9,258 (7.3)	164-2 (5.8)	266-2 (4.8)	519-2 (3.7)	887-2 (3.0)	296-3 (1.8)	106-4 (1.1)

$(t/\mu) \times 100$ ratios in parentheses are for P(A) = .95 (or more)

The figure following the dash in sample size numbers shows the number of zeros to add; for example, 296-3 = 296,000.

TABLE 4g

Table of Sampling Plans for $\beta = 3\ 1/3$

c	n												
	(t/μ) x 100 Ratio for which P(A) = .10 (or less)												
	100	65	50	40	30	25	20	15	12	10	8	6.5	5
0	4 (30)	14 (21)	34 (16)	70 (13)	183 (9.6)	334 (8.0)	698 (6.4)	1,772 (4.8)	3,839 (3.8)	6,979 (3.2)	149-2 (2.5)	288-2 (2.1)	698-2 (1.6)
1	7 (46)	24 (32)	57 (24)	119 (19)	309 (14)	564 (12)	1,179 (9.8)	2,993 (7.4)	6,484 (5.8)	118-2 (4.8)	251-2 (3.9)	486-2 (3.2)	118-3 (2.4)
2	9 (56)	33 (37)	78 (28)	164 (23)	423 (17)	772 (14)	1,613 (11)	4,094 (8.7)	8,870 (6.8)	161-2 (5.7)	343-2 (4.5)	665-2 (3.7)	161-3 (2.8)
3	12 (60)	42 (40)	98 (31)	206 (25)	531 (19)	969 (15)	2,025 (12)	5,140 (9.4)	111-2 (7.5)	202-2 (6.2)	431-2 (4.9)	835-2 (4.0)	202-3 (3.1)
4	14 (65)	51 (42)	117 (33)	246 (26)	635 (20)	1,159 (16)	2,423 (13)	6,150 (10)	133-2 (7.9)	242-2 (6.6)	516-2 (5.2)	999-2 (4.3)	242-3 (3.3)
5	16 (68)	59 (44)	136 (34)	286 (27)	737 (21)	1,345 (17)	2,811 (14)	7,135 (10)	155-2 (8.2)	281-2 (6.8)	598-2 (5.4)	116-3 (4.4)	281-3 (3.4)
6	19 (69)	67 (46)	157 (35)	325 (28)	836 (21)	1,527 (17)	3,192 (14)	8,102 (11)	176-2 (8.4)	319-2 (7.0)	679-2 (5.6)	132-3 (4.6)	319-3 (3.5)
7	21 (71)	75 (47)	176 (36)	363 (29)	935 (22)	1,698 (18)	3,567 (14)	9,055 (11)	196-2 (8.7)	357-2 (7.2)	759-2 (5.8)	147-3 (4.7)	357-3 (3.6)
8	23 (73)	83 (47)	194 (36)	400 (29)	1,032 (22)	1,884 (18)	3,938 (15)	9,997 (11)	217-2 (8.8)	394-2 (7.4)	838-2 (5.9)	162-3 (4.8)	394-3 (3.7)
9	26 (74)	90 (48)	212 (37)	438 (30)	1,128 (22)	2,059 (19)	4,305 (15)	109-2 (11)	237-2 (9.0)	430-2 (7.5)	917-2 (6.0)	178-3 (4.9)	430-3 (3.7)
10	28 (75)	101 (48)	230 (38)	475 (30)	1,228 (23)	2,233 (19)	4,669 (15)	119-2 (12)	257-2 (9.2)	467-2 (7.6)	994-2 (6.0)	193-3 (4.9)	467-3 (3.8)
11	30 (76)	108 (49)	247 (38)	511 (31)	1,318 (23)	2,406 (19)	5,030 (15)	128-2 (12)	277-2 (9.3)	503-2 (7.7)	107-3 (6.1)	207-3 (5.0)	503-3 (3.8)
12	32 (77)	116 (49)	265 (39)	548 (31)	1,412 (23)	2,578 (20)	5,389 (15)	137-2 (12)	296-2 (9.4)	539-2 (7.8)	115-3 (6.2)	222-3 (5.0)	539-3 (3.9)
13	35 (77)	124 (50)	283 (39)	584 (31)	1,505 (24)	2,748 (20)	5,745 (16)	146-2 (12)	316-2 (9.4)	574-2 (7.9)	122-3 (6.2)	237-3 (5.1)	574-3 (3.9)
14	37 (78)	131 (50)	300 (39)	620 (32)	1,598 (24)	2,918 (20)	6,100 (16)	155-2 (12)	335-2 (9.5)	610-2 (8.0)	130-3 (6.3)	252-3 (5.2)	610-3 (3.9)
15	39 (79)	139 (51)	317 (40)	656 (32)	1,690 (24)	3,086 (20)	6,453 (16)	164-2 (12)	355-2 (9.6)	645-2 (8.0)	137-3 (6.4)	266-3 (5.2)	645-3 (4.0)

(t/μ) x 100 ratios in parentheses are for P(A) = .95 (or more)

The figure following the dash in sample size numbers shows the number of zeros to add; for example, 319-3 = 319,000.

TABLE 4h

Table of Sampling Plans for $\beta = 4$

c	n												
	(t/μ) x 100 Ratio for which P(A) = .10 (or less)												
	100	80	65	50	40	30	25	20	15	12	10	8	6.5
0	4 (37)	9 (30)	20 (25)	55 (19)	134 (15)	419 (12)	886 (9.4)	2,094 (7.6)	6,774 (5.8)	164-2 (4.7)	329-2 (4.0)	768-2 (3.2)	177-3 (2.6)
1	7 (53)	15 (44)	33 (36)	93 (27)	228 (22)	708 (16)	1,497 (14)	3,537 (11)	114-2 (8.0)	278-2 (6.5)	556-2 (5.6)	130-3 (4.5)	299-3 (3.7)
2	9 (62)	21 (50)	46 (40)	128 (31)	312 (25)	968 (19)	2,047 (16)	4,839 (12)	157-2 (9.2)	380-2 (7.4)	760-2 (6.2)	177-3 (5.1)	409-3 (4.2)
3	12 (66)	26 (54)	57 (44)	162 (33)	391 (27)	1,215 (20)	2,570 (17)	6,074 (13)	197-2 (10)	477-2 (7.9)	954-2 (6.6)	223-3 (5.5)	514-3 (4.5)
4	15 (68)	31 (56)	69 (46)	194 (35)	468 (28)	1,454 (21)	3,075 (18)	7,268 (14)	235-2 (10)	571-2 (8.3)	114-3 (7.0)	266-3 (5.8)	615-3 (4.7)
5	17 (71)	37 (58)	80 (48)	225 (36)	543 (29)	1,687 (22)	3,568 (18)	8,432 (15)	273-2 (11)	663-2 (8.6)	133-3 (7.2)	309-3 (6.0)	713-3 (4.9)
6	19 (74)	42 (59)	91 (49)	255 (37)	616 (30)	1,915 (22)	4,051 (19)	9,575 (15)	310-2 (11)	752-2 (8.8)	150-3 (7.4)	351-3 (6.1)	810-3 (5.0)
7	22 (75)	47 (60)	102 (50)	286 (38)	689 (30)	2,140 (23)	4,528 (19)	107-2 (15)	346-2 (11)	841-2 (9.0)	168-3 (7.6)	392-3 (6.2)	905-3 (5.1)
8	24 (76)	52 (61)	112 (50)	315 (39)	760 (31)	2,363 (23)	4,998 (19)	118-2 (15)	382-2 (12)	928-2 (9.1)	186-3 (7.7)	433-3 (6.3)	100-4 (5.2)
9	27 (77)	57 (62)	123 (51)	344 (39)	831 (31)	2,583 (23)	5,464 (20)	129-2 (16)	418-2 (12)	101-3 (9.2)	203-3 (7.8)	474-3 (6.3)	109-4 (5.3)
10	29 (78)	64 (62)	136 (51)	373 (40)	901 (32)	2,802 (24)	5,926 (20)	140-2 (16)	453-2 (12)	110-3 (9.4)	220-3 (7.8)	514-3 (6.4)	119-4 (5.3)
11	31 (79)	69 (63)	147 (52)	402 (40)	971 (32)	3,018 (24)	6,384 (20)	151-2 (16)	488-2 (12)	119-3 (9.4)	237-3 (7.9)	553-3 (6.4)	128-4 (5.4)
12	33 (80)	74 (63)	157 (52)	431 (40)	1,040 (32)	3,233 (24)	6,840 (20)	162-2 (16)	523-2 (12)	127-3 (9.5)	254-3 (8.0)	593-3 (6.5)	137-4 (5.4)
13	36 (81)	79 (64)	167 (53)	460 (41)	1,109 (33)	3,447 (24)	7,292 (20)	172-2 (16)	558-2 (12)	135-3 (9.6)	271-3 (8.1)	632-3 (6.5)	146-4 (5.5)
14	38 (82)	84 (64)	178 (53)	488 (41)	1,177 (33)	3,660 (25)	7,742 (21)	183-2 (17)	592-2 (12)	144-3 (9.7)	288-3 (8.2)	671-3 (6.6)	155-4 (5.5)
15	40 (82)	89 (64)	188 (53)	516 (41)	1,246 (33)	3,872 (25)	8,190 (21)	194-2 (17)	626-2 (12)	152-3 (9.8)	304-3 (8.2)	710-3 (6.6)	164-4 (5.6)

(t/μ) x 100 ratios in parentheses are for P(A) = .95 (or more)

The figure following the dash in sample size numbers shows the number of zeros to add; for example, 304-3 = 304,000.

TABLE 4i

Table of Sampling Plans for β = 5

c	n												
	(t/μ) x 100 Ratio for which P(A) = .10 (or less)												
	100	80	65	50	45	40	35	30	25	20	15	12	10
0	4 (46)	11 (37)	31 (30)	113 (23)	192 (21)	344 (19)	678 (16)	1,440 (14)	3,599 (12)	110-2 (9.4)	461-2 (7.0)	135-3 (5.6)	329-3 (4.8)
1	7 (53)	19 (49)	53 (40)	193 (31)	325 (28)	581 (25)	1,145 (22)	2,432 (19)	6,079 (15)	185-2 (12)	778-2 (9.2)	229-3 (7.5)	556-3 (6.3)
2	10 (68)	26 (55)	72 (45)	264 (34)	444 (31)	795 (27)	1,566 (24)	3,327 (21)	8,316 (17)	253-2 (14)	106-3 (10)	313-3 (8.3)	760-3 (7.0)
3	12 (73)	33 (58)	90 (47)	331 (36)	557 (33)	998 (29)	1,965 (25)	4,176 (22)	104-2 (18)	318-2 (14)	134-3 (11)	393-3 (8.8)	954-3 (7.4)
4	15 (76)	40 (60)	108 (49)	396 (38)	667 (34)	1,194 (30)	2,352 (26)	4,997 (23)	125-2 (19)	381-2 (15)	160-3 (11)	470-3 (9.1)	114-4 (7.6)
5	17 (78)	46 (62)	125 (51)	460 (39)	773 (35)	1,385 (31)	2,728 (27)	5,797 (23)	145-2 (19)	442-2 (15)	186-3 (12)	546-3 (9.4)	132-4 (7.8)
6	20 (79)	53 (63)	143 (52)	522 (40)	878 (36)	1,572 (32)	3,098 (28)	6,583 (24)	165-2 (20)	502-2 (16)	211-3 (12)	620-3 (9.6)	150-4 (8.0)
7	22 (80)	59 (64)	162 (52)	583 (40)	981 (36)	1,757 (32)	3,463 (28)	7,357 (24)	184-2 (20)	561-2 (16)	235-3 (12)	692-3 (9.7)	168-4 (8.1)
8	25 (81)	66 (65)	179 (53)	644 (41)	1,083 (37)	1,940 (33)	3,823 (29)	8,122 (25)	203-2 (20)	619-2 (16)	260-3 (12)	764-3 (9.8)	187-4 (8.2)
9	27 (82)	72 (66)	195 (54)	704 (41)	1,184 (37)	2,121 (33)	4,179 (29)	8,879 (25)	222-2 (20)	676-2 (16)	284-3 (12)	836-3 (10)	203-4 (8.3)
10	30 (83)	81 (66)	212 (54)	763 (41)	1,284 (37)	2,300 (33)	4,532 (29)	9,630 (25)	241-2 (21)	734-2 (17)	308-3 (12)	906-3 (10)	220-4 (8.4)
11	32 (84)	87 (66)	228 (55)	822 (42)	1,384 (38)	2,478 (34)	4,882 (29)	104-2 (25)	259-2 (21)	790-2 (17)	332-3 (13)	976-3 (10)	237-4 (8.5)
12	34 (84)	93 (67)	244 (55)	881 (42)	1,482 (38)	2,655 (34)	5,230 (30)	111-2 (25)	278-2 (21)	847-2 (17)	356-3 (13)	105-4 (10)	254-4 (8.6)
13	37 (85)	99 (67)	261 (55)	939 (42)	1,580 (38)	2,830 (34)	5,576 (30)	118-2 (26)	296-2 (21)	903-2 (17)	379-3 (13)	112-4 (10)	271-4 (8.6)
14	39 (85)	105 (68)	277 (56)	997 (43)	1,678 (39)	3,005 (34)	5,920 (30)	126-2 (26)	314-2 (21)	958-2 (17)	403-3 (13)	118-4 (10)	288-4 (8.6)
15	41 (86)	111 (68)	293 (56)	1,055 (43)	1,775 (39)	3,178 (35)	6,263 (30)	133-2 (26)	333-2 (21)	101-3 (17)	426-3 (13)	125-4 (10)	304-4 (8.7)

(t/μ) x 100 ratios in parentheses are for P(A) = .95 (or more)

The figure following the dash in sample size numbers shows the number of zeros to add; for example, 203-2 = 20,300.

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